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# Evaluation of aerodynamic criteria in the design of a small wind turbine with the lifting line model

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## Abstract

This thesis deals with the optimization of the aerodynamic design of a small wind turbine to maximize energy production.

It is well known that energy production depends on the type of turbine and on the wind characteristics on a given site. In this work we try to maximize the energy production for a given site by optimizing the most important aerodynamic design parameters.

The optimization is carried out with computer codes for the design and analysis of a horizontal axis wind turbine based on lifting line theory. The starting point is a 3-bladed, 2.5 meter diameter variable speed turbine design. The main parameters to be optimized are the type airfoil of blade sections together with its design lift coefficient. The selection of the airfoil is based on the lift and drag data as function of angle of attack at different Reynolds numbers. In particular, the curves of  $C_L/C_D$  are used to select the design parameters lift coefficient. Other parameters concern the design wind speed and the tip speed ratio (TSR). The diameter and the number of blades are kept constant in the optimization. A structural constraint for maximum stress on the blades is used to define the rated power.

The variant designs are evaluated by determining their annual energy production (AEP) for two representative sites.

## 1. Introduction

Wind energy has always played an important role in our society. 3000 years ago already, humans employed this energy for grinding grain or pumping water. Nowadays, for environmental reasons, we are using wind to produce “green energy”. The purpose is also that homes, farms, small businesses, schools, and other institutions throughout the country use small wind turbines (SWT) to lower or eliminate their electricity bills [1]. In fact, SWTs are more accessible to everyone by their small sizes and reasonable costs while the largest turbines are massive and expensive.

## 2. Aerodynamic design criteria

In trying to improve the power, the first step is to select the best design criteria to set the characteristics of wind turbine. The Table 2.1 presents the aerodynamic design parameters chosen. Some parameters are kept fixed and have been determined based on a previous project of Pôtra [2]. The following parameters were varied according to different criteria:

- *The angle of attack*: it varies along the blade to give the optimum  $C_L/C_D$ . In the previous project, the angle of attack was kept constant.
- *The choice of airfoil*: Airfoil will be different to try to improve the power.

Table 2.1: Aerodynamic Design Parameters.

Number of blades	3
Rotor diameter	2.5 m
Hub diameter	0.375 m
Tip speed ratio	7
Wind speed	10 m/s
Angle of attack ( $\alpha$ )	Optimum $C_L/C_D$
Airfoil	To be chosen

The wind speed  $U$  of 10m/s and the Tip Speed Ratio TSR of 7, chosen for the designs, determine the velocity triangle geometry. In consequence, the aerodynamic pitch angle  $\beta$  will be fixed for each radius on the blade.

The velocity triangle is represented in Fig. 2.1 and is composed by  $\omega r$  and  $U$ .

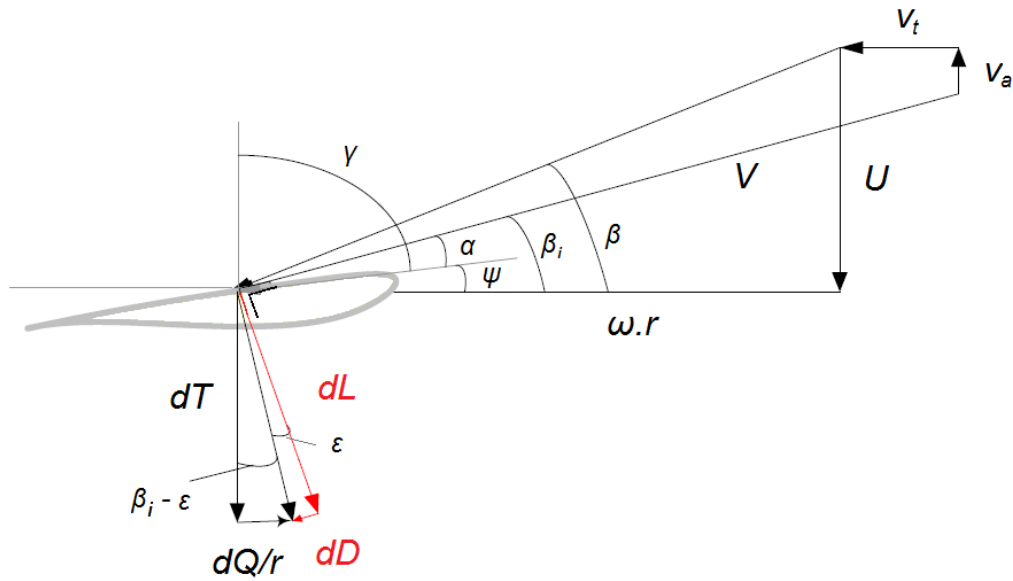


Fig. 2.1: Velocities triangle and forces on a blade section

Moreover, as the power of SWTs is generally controlled by stall regulation, the geometry is fixed. By this way, the pitch angle  $\psi$  will be constant for each radius along the blade. So the angle of attack will depend only on the aerodynamic induced pitch angle  $\beta_i$ .

All the calculations of this thesis are based on the lifting line theory. For a more detailed account, we refer to [3].

### 3. Candidate airfoils

One of the important design criteria is the choice of the airfoil. Two airfoils have been selected:

- *Eppler E387*
- *Wortmann FX 63-137*

The base airfoils of *E387* and *FX 63-137* are depicted in the Fig. 3.1. Note that the maximum thickness of *FX 63-137* represents 13.63 % of the chord while *E387* has a maximum thickness of 9.06 % chord. Inspection of the airfoil shapes shows that the airfoil *FX 63-137* has more camber than *E387*, and this difference gives a higher lift coefficient at the same angle of attack.

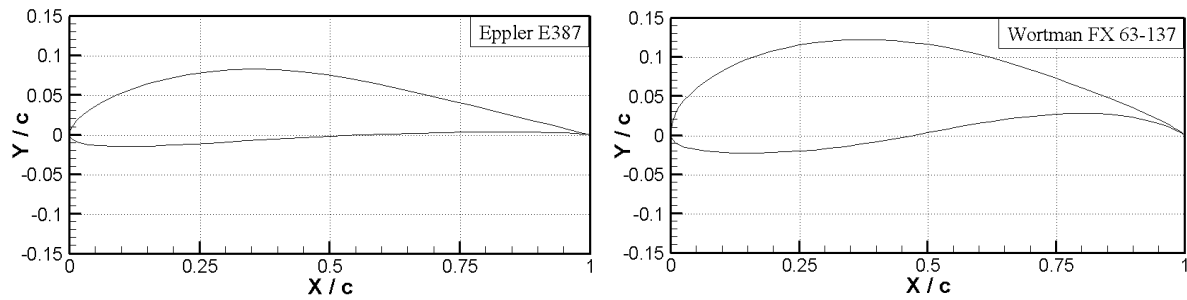


Fig. 3.1: E387 and FX 63-137 base airfoils.

Selected design criteria will be applied to these airfoils and results will be compared to determine the best one which will be used.

#### 4. Comparison between designs at constant angle of attack and Optimum $C_L/C_D$

The first comparison concerns two different design options:

- *Constant angle of attack*: the angle of attack is kept constant along the blades. This method has been used in [2].
- *Optimum  $C_L/C_D$* : the angle of attack varies along the blades to give the optimum conditions.

This comparison allows seeing if the new method, *Optimum  $C_L/C_D$* , gives a better power coefficient.

Table 4.1 shows the values of the power coefficient  $C_P$  and thrust coefficient  $C_T$  for both designs.

The comparison of the results indicates a slight increase of  $C_P$  and  $C_T$  with the design of *Optimum  $C_L/C_D$* .

Table 4.1: Power and Thrust coefficient for both design methods.

	$C_T$	$C_P$
Constant angle of attack	0.777	0.463
Optimum $C_L/C_D$	0.779	0.467

#### 5. Comparison of airfoils for design method at optimum $C_L/C_D$

The second comparison concerns the design at *Optimum  $C_L/C_D$*  applied to:

- *The airfoil Eppler E387*
- *The airfoil Wortmann FX 63-137*

Applying the best design, design of *Optimum  $C_L/C_D$* , this comparison shows which airfoil will lead to a higher turbine power.

Table 5.1 shows the values of the power coefficient  $C_P$  and thrust coefficient  $C_T$  of *E387* and *FX 63-137* for design at *Optimum  $C_L/C_D$* .

The comparison of the results indicates a decrease of  $C_P$  for airfoil *FX 63-137*. Finally, the best airfoil is *E387*.

Table 5.1: Power and Thrust coefficient for both airfoils.

	$C_T$	$C_P$
E387	0.779	0.467
FX 63-137	0.78	0.457

We have shown that the best design is achieved with *E387* with *Optimum  $C_L/C_D$* . At present, we can determine the power curve for this design in order to calculate the Annual Energy Production (AEP). However, it is interesting to compare the AEP for both airfoils at *Optimum  $C_L/C_D$*  to draw conclusions. Therefore, the AEP calculations are done for *E387* and *FX 63-137*.

## 6. Power curves

To calculate the energy production of wind turbines, the power as function of wind speed, namely the power curve, is necessary.

For each TSR, we obtain different powers at each wind speed, so each TSR represents a different power curve. The purpose of this step is to determine the value of TSR that maximizes the power.

Unfortunately, as the power formula is very much influenced by the wind speed, we have only to plot the power coefficient  $C_P$  versus wind speed.

Fig. 6.1 shows power coefficient graphs as function of wind speed. This allows getting curves with better precision and simplifying the choice of optimum TSR to give maximum power.

As the power depends on the cube of the wind speed, large wind speeds are more important than small ones. Therefore, for the calculations, we consider only TSRs giving the best  $C_P$  from a wind speed of 7 m/s. In fact, the best TSRs before this wind speed are not interesting to study because they have a small influence on the power due to the small wind speed.

So, four cases will be considered:

- *Airfoil Eppler E387 with TSR = 6.75, renamed "Ep-6.75".*
- *Airfoil Eppler E387 with TSR = 7, renamed "Ep-7".*

- Airfoil Wortmann FX 63-137 with TSR = 7 renamed “Wo-7”.
- Airfoil Wortmann FX 63-137 with TSR = 7.25, renamed “Wo-7.25”.

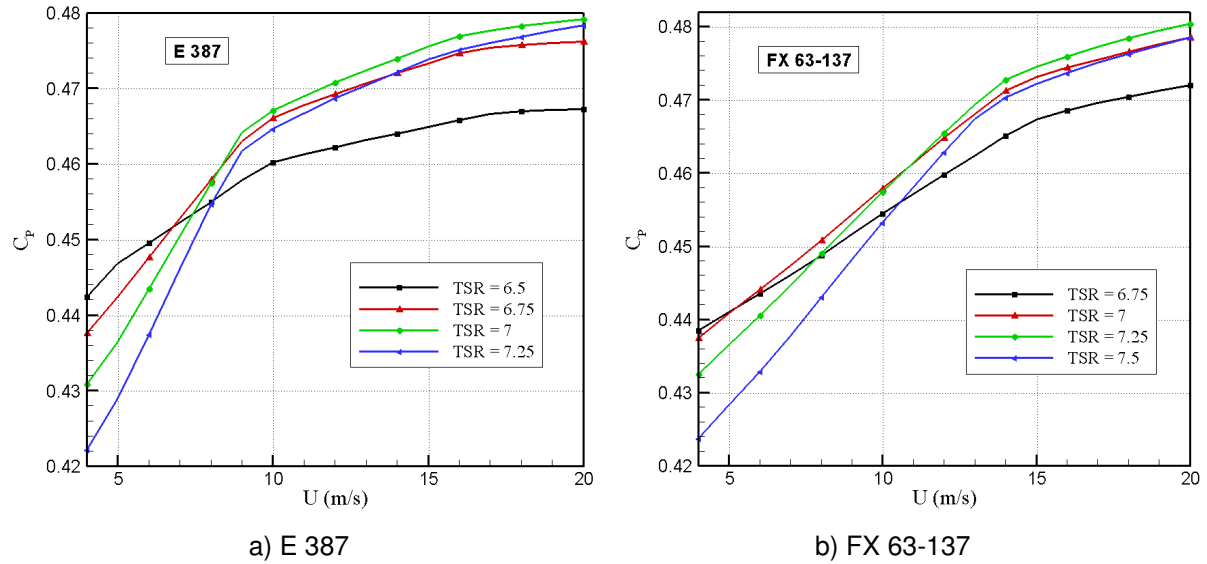


Fig. 6.1: Power coefficient curves for E387 and FX 63-137.

## 7. Stress analysis

After the TSRs selection, we have to determine the rated wind speed which depends on the stress limit and thus on the blades material.

The material chosen for the blades is Polycarbonate reinforced by carbon fibers, and so the limit stress value assumed, including a safety factor [4], is:

$$\sigma_{lim} = \frac{R_e}{s} = \frac{200}{1.35} \approx 150 \text{ MPa}$$

where  $R_e$  is the elastic limit and  $s$  is the safety factor.

The stresses along the blades calculated will have to be lower than this limit. So, for selected TSRs chosen before, the rated wind speed will be determined and the stress will not exceed the limit stress value.

For *E387*, we find that the maximum wind speed admitted (rated wind speed) is 18 m/s for both selected TSRs. While for *FX 63-137* with selected TSRs, the maximum admissible stress occurs for a wind speed of 13 m/s.

Until the rated wind speed, the wind turbine rotor will have to operate at variable rotational velocity to keep a constant TSR. Then the TSR and the rotational velocity decrease to keep a constant power until the cut-out wind speed, the speed at which the turbine is shut down.

Fig. 7.1 shows the comparison between the power curves of *Ep-7* and *Wo-7* with the rated wind speeds found before. The cut-out wind speed is assumed equal to 25 m/s. On the graph, we can see that the power of generators used by the wind turbines is not the same. In fact, for *Ep-7* it reaches around 8 kW while only 3 kW for *Wo-7*. As a consequence, it is meaningless to compare the Annual Energy Production with two different generators. Of course, the largest generator will produce more energy on windy sites.

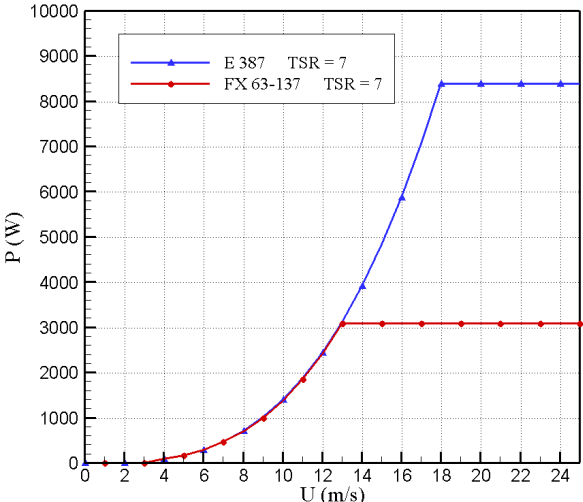


Fig. 7.1: Power curves with unreal cut-out wind speed.

The rated wind speed of *Ep-6.75* and *Ep-7* (18 m/s) is too large. To be more realistic and to get a fair comparison between airfoils, we set the rated wind speed at 13 m/s. In that case, the same class of generators (about 3 kW) will be obtained for *Ep-6.75*, *Ep-7*, *Wo-7* and *Wo-7.25*, and a better comparison will be made.

Fig. 7.2 shows the real power curves for *E387* and *FX 63-137*. Note that the curves for selected TSRs are almost the same but the difference cannot be noticed on these graphs.

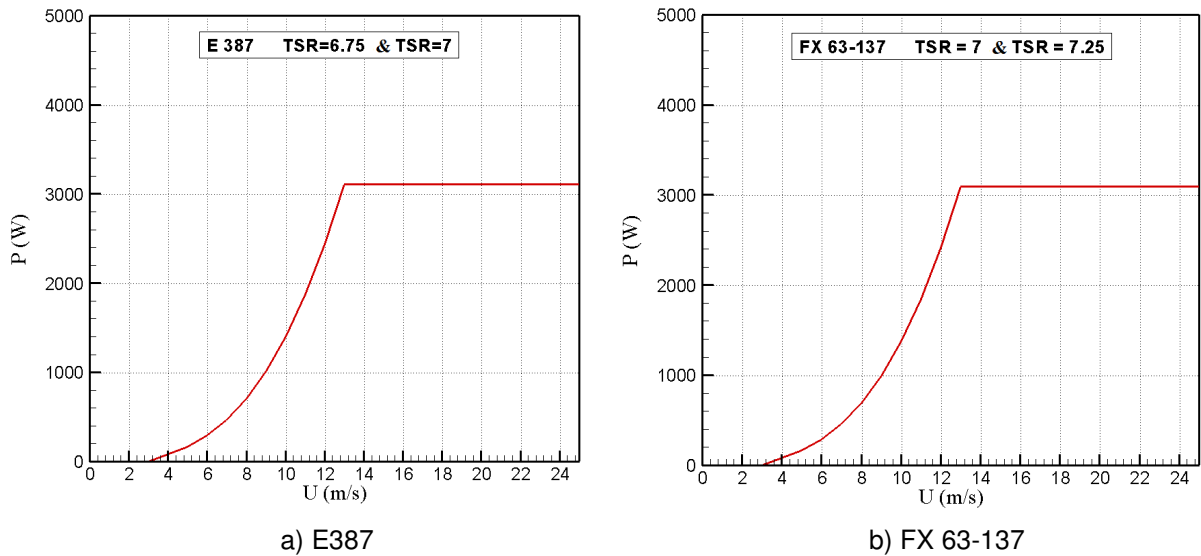


Fig. 7.2: Real power curves for E387 and FX 63-137 with selected TSRs.

## 8. Sites description

To calculate the Annual Energy production AEP, two different sites are selected, a weakly windy site and a strongly windy site:

- *Montijo*

This location is situated in Lisbon. The mean wind speed reaches only 4.09 m/s. The scale parameter  $c$  of the Weibull distribution is 4.7 m/s whereas the shape parameter  $k$  is 2.01. The wind turbine is placed at an altitude of 11m and the tower height is 10m.

- *Picarreira*

This site is situated in Vila Real in the north of Portugal. Picarreira is very windy and the mean wind speed is around 6.38 m/s. Concerning the Weibull distribution, the scale parameter  $c$  is 7.3 m/s and the shape parameter  $k$  is 1.75. The wind turbine is installed at an altitude of 1056 m and the tower height is 30 m.

## 9. Weibull distributions

Fig. 9.1 shows the Weibull distributions of the selected sites. Concerning Montijo, note that frequency percentage is very high at low wind speeds but falls down quickly to reach zero around 12 m/s. In Picarreira, the frequency percentage is not so high at low wind speeds, which allows getting a curve falling down slowly to reach zero around 19 m/s.

As the power depends on the cube of wind speed, a small frequency for high wind speeds is more important than a high frequency for small wind speeds. As a consequence, a wind turbine placed in Picarreira will produce more power than in Montijo.



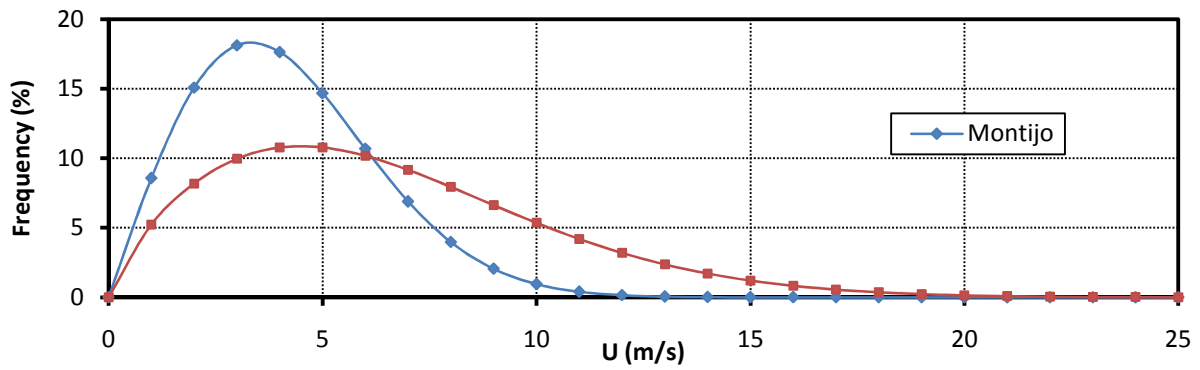


Fig. 9.1: Weibull distribution of selected sites.

## 10. Annual energy production

In Montijo,  $Ep-6.75$  allows getting an Annual Energy Production of 1562.99 kWh against 1554.92 kWh for  $Ep-7$ , as shown in Table 10.1. However, the difference represents only 0.5 %. Concerning FX 63-137,  $Wo-7$  produces 1546.01 kWh against 1537.15 kWh for  $Wo-7.25$ . This difference is 0.6 %.

Generally, as expected previously, the comparison of the AEP results indicates that  $E387$  produces more electricity than  $FX\ 63-137$ . However, when the best results for each airfoil are compared, the AEP of  $Ep-6.75$  is only 1.1 % larger than  $Wo-7$ .

In Picarreira, the Annual Energy Production reaches 6071.88 kWh for  $Ep-7$  against 6066.01 kWh for  $Ep-6.75$ . The difference represents only 0.1%.

Concerning FX 63-137, the energy production for  $Wo-7$  is 6001.19 kWh per year and 5997.2 kWh for  $Wo-7.25$ . This difference is 0.1 %.

At present, when the best results of both airfoils are compared in Picarreira, the AEP of  $Ep-7$  is only 1.2 % larger than  $Wo-7$ .

Table 10.1: Annual Energy Production.

Airfoil	Annual Energy Production (kWh)	
	Montijo	Picarreira
E387 with TSR=6.75	1562.99	6066.01
E387 with TSR=7	1554.92	6071.88
FX63-137 with TSR=7	1546.01	6001.19
FX63-137 with TSR=7.25	1537.15	5997.2

Note that the difference of Annual Energy Production between Montijo and Picarreira is very large. Indeed, for each selected TSRs for both airfoils, the increase reaches around 290%. Note also that the

best TSR for *E387* has changed between these two sites. In Montijo, the best design is *Ep-6.75* while in Picarreira it is *Ep-7*.

## 11. Conclusion

The purpose of the project was the improvement of output electrical power for a small wind turbine through the modification of the design parameters, in particular the airfoil and the design angle of attack.

The following conclusions can be drawn:

- The choice of an “optimum” angle of attack for the maximization of  $C_L/C_D$  allows getting better results.
- The airfoil *E387* obtains a better power coefficient than Wortmann FX 63-137.
- The wind turbine using Eppler *E387* produces more energy than FX 63-137 for all sites analyzed.
- The optimum TSR for Eppler *E387* changes as function of the site.
- The site has a strong influence on the Annual Energy Production.
- Concerning the AEP of a small wind turbine, the modification of the design criteria enables only very small gains.
- The aerodynamic design optimization may be less important for a small wind turbine than for a large turbine.

## 12. Bibliography

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